Definition 3.7

Two sets A and B are said to be equivalent, written as A↔B (or A~B), if there is a one-to-one correspondence between them.

Observe that two finite sets A and B are equivalent, if and only if

$n(A) = n(B)$

Example 5 Let $A = \{\sqrt{2}, e, \pi\}$ and $B = \{1, 2, 3\}$.

Since $n(A) = n(B)$, A and B are equivalent sets and we write

$A \leftrightarrow B$.

Note that equal sets are always equivalent since each element can be matched with itself, but equivalent sets are not necessarily equal. For example,

$\{1, 2\} \leftrightarrow \{a, b\}$ but $\{1, 2\} \neq \{a, b\}$.

Exercise 3.5

Which of the following pairs represent equal sets and which of them represent equivalent sets?

1. $\{a, b\}$ and $\{2, 4\}$
2. $\{\varnothing\}$ and $\varnothing$
3. $\{x \in \mathbb{N} \mid x < 5\}$ and $\{2, 3, 4, 5\}$
4. $\{1, \{2, 4\}\}$ and $\{1, 2, 4\}$
5. $\{x \mid x < x\}$ and $\{x \in \mathbb{N} \mid x < 1\}$

3.3 OPERATIONS ON SETS

There are operations on sets as there are operations on numbers. Like the operations of addition and multiplication on numbers, intersection and union are operations on sets.

3.3.1 Union, Intersection and Difference of Sets

A Union of sets

Definition 3.8

The union of two sets A and B, denoted by A∪B and read "A union B" is the set of all elements that are members of set A or set B or both of the sets. That is, $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

The *red* shaded region of the diagram in the figure on the right represents A ∪ B.

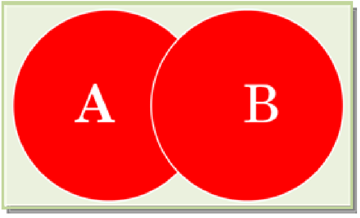


Figure 3.7

An element common to both sets is listed only once in the union. For example, if $A = \{a, b, c, d, e\}$ and $B = \{c, d, e, f, g\}$, then

$A \cup B = \{a, b, c, d, e, f, g\}$.

Example 1

a $\{a, b\} \cup \{c, d, e\} = \{a, b, c, d, e\}$

b $\{1, 2, 3, 4, 5\} \cup \varnothing = \{1, 2, 3, 4, 5\}$

Properties of the union of sets

ACTIVITY 3.10

Let $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$.

1 Find **a** $A \cup B$ **b** $B \cup A$

What is the relationship between $A \cup B$ and $B \cup A$?

2 Find **a** $A \cup B$ **b** $(A \cup B) \cup C$ **c** $B \cup C$ **d** $A \cup (B \cup C)$

What is the relationship between $(A \cup B) \cup C$ and $A \cup (B \cup C)$?

3 Find $A \cup \varnothing$, what is the relationship between $A \cup \varnothing$ and A?

The above Activity leads you to the following properties:

For any sets A, B and C

1 Commutative property $A \cup B = B \cup A$

2 Associative property $(A \cup B) \cup C = A \cup (B \cup C)$

3 Identity property $A \cup \varnothing = A$

Exercise 3.6

1 Given $A = \{1, 2, \{3\}\}$, $B = \{2, 3\}$ and $C = \{\{3\}, 4\}$, find:

a $A \cup B$ **b** $B \cup C$ **c** $A \cup C$

d $A \cup (B \cup C)$ **e** $(A \cup B) \cup C$

2 State whether each of the following statements is true or false:

a If $x \in A$ and $x \notin B$, then $x \notin (A \cup B)$.

b If $x \in (A \cup B)$ and $x \notin A$, then $x \in B$.

c If $x \notin A$ and $x \notin B$, then $x \notin (A \cup B)$.

d For any set A, $A \cup A = A$.

e For any set A, $A \cup \varnothing = A$.

f If $A \subseteq B$, then $A \cup B = B$.

g For any two sets A and B, $A \subseteq (A \cup B)$ and $B \subseteq (A \cup B)$.

h For any three sets A, B and C, if $A \subseteq B$, and $B \subset C$, then $A \cup B = C$.

i For any three sets A, B and C, if $A \cup B = C$, then $B \subset C$.

j If $A \cup B = \varnothing$, then $A = \varnothing$ and $B = \varnothing$.

3 Using copies of the Venn diagrams below, shade $A \cup B$.

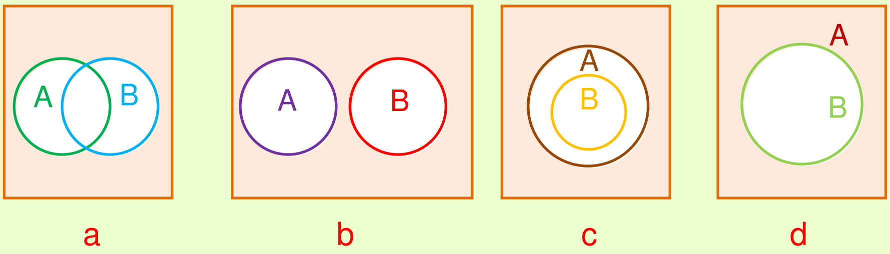


Figure 3.8

B Intersection of sets

ACTIVITY 3.11

Consider the two sets G = {2, 4, 6, 8, 10, 12} and H = {1, 2, 3, 4, 5}.

a Draw a Venn diagram that shows the relationship between the two sets.

b Shade the region common to both sets and find their common elements.

Definition 3.9

The intersection of two sets A and B, denoted by $A \cap B$ and read as "*A intersection B*", is the set of all elements common to both set A and set B. That is, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

Using the Venn diagram, $A \cap B$ is represented by the *blue* shaded region:

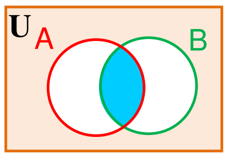


Figure 3.9

Example 2 Let S = {*a*, *b*, *c*, *d*} and T = {*f*, *b*, *d*, *g*}. Then $S \cap T$ = {*b*, *d*}.

Example 3 Let V = {2, 4, 6,...} (multiples of 2) and

W = {3, 6, 9,...} (multiples of 3).

Then $V \cap W$ = {6, 12, 18,...}, that is, multiples of 6.

Example 4 Let A = {1, 2, 3} and B = {5, 6, 7, 8}, then $A \cap B = \varnothing$.

Definition 3.10

Two or more sets are disjoint if they have no common element.

A and B are disjoint, if and only if $A \cap B = \varnothing$.

In the Venn diagram, the sets A and B are disjoint.

Here $A \cap B = \varnothing$

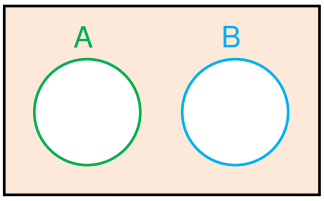


Figure 3.10

Properties of the intersection of sets

ACTIVITY 3.12

Let U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} be the universal set and let

A = {0, 2, 3, 5, 7}, B = {0, 2, 4, 6, 8} and

$C = \{x \mid x \text{ is a factor of } 6\}$

1 Find **a** $A \cap C$ **b** $C \cap A$

What is the relationship between $A \cap C$ and $C \cap A$?

2 Find **a** $A \cap B$ **b** $(A \cap B) \cap C$ **c** $B \cap C$ **d** $A \cap (B \cap C)$

What is the relationship between $(A \cap B) \cap C$ and $A \cap (B \cap C)$?

3 Find $A \cap U$. What is the relationship between $A \cap U$ and A?

The above Activity leads you to the following properties:

For any sets A, B and C and the universal set U

1 Commutative Property: $A \cap B = B \cap A$.

2 Associative Property: $(A \cap B) \cap C = A \cap (B \cap C)$.

3 Identity Property: $A \cap U = A$.

Exercise 3.7

1 Given $A = \{a, b, \{c\}\}$, $B = \{b, c\}$ and $C = \{\{c\}, d\}$, find:

a $A \cap B$ **b** $A \cap C$ **c** $B \cap C$ **d** $A \cap (B \cap C)$

2 State whether each of the following statements is true or false:

a If $x \in A$ and $x \notin B$, then $x \in (A \cap B)$.

b If $x \in (A \cap B)$, then $x \in A$ and $x \in B$.

c If $x \notin A$ and $x \in B$, then $x \in (A \cap B)$.

d For any set A, $A \cap A = A$.

e If $A \subseteq B$, then $A \cap B = A$.

f For any two sets A and B, $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

g If $A \cap B = \varnothing$, then $A = \varnothing$ or $B = \varnothing$.

h If $(A \cup B) \subseteq A$, then $B \subseteq A$.

i If $A \subseteq B$, then $A \cap B = B$.

j If $A \subseteq B$, then $A \cap B = \varnothing$.

k If $A \subseteq B$, then $B' \subseteq A'$.

3 In each Venn diagram below, shade $(A \cap B) \cap C$.

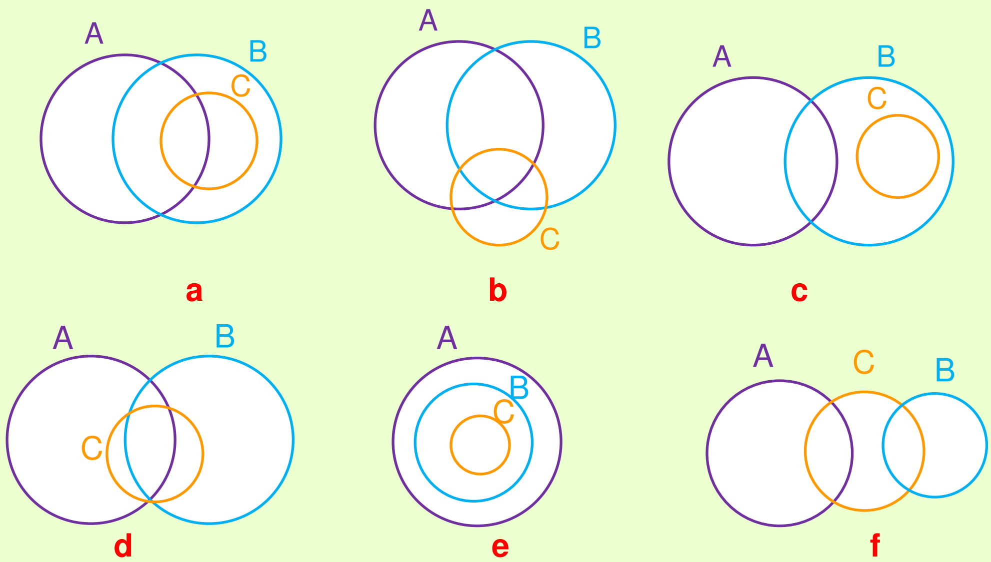


Figure 3.11

C Difference and symmetric difference of sets

i The relative complement (or difference) of two sets

Given two sets A and B, the complement of B relative to A (or the difference between A and B) is defined as follows.

Definition 3.11

The relative complement of a set B with respect to a set A (or the difference between A and B), denoted by A – B, read as "A difference B", is the set of all elements in A that are not in B.

That is, $A - B = \{x \mid x \in A \text{ and } x \notin B\}$.

Note: A – B is sometimes denoted by A\B. (read as "A less B")

A – B and A\B are used interchangeably.

Using a Venn diagram, A\B can be represented by shading the region in A which is not part of B.

A\B is shaded in *light green*.

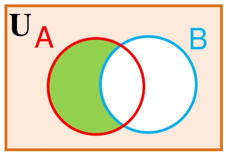


Figure 3.12

Example 5 If A = {x, y, z, w} and B = { a, b, x, y}, then find:

a the complement of B relative to A **b** B\ A **c** B\B

Solution:

a Note that finding "the complement of B relative to A" is the same as finding "the relative complement of B with respect to A". That is A\B.

So, A\B = {z, w}.

b B\A = { a, b}.

c B\B = ∅.

ACTIVITY 3.13

Let A = {0, 2, 3, 5, 7}, B = {0, 2, 4, 6, 8} and C = {1, 2, 3, 6}. Find:

a A\B **b** B\A **c** (A\B)\C **d** A\(B\C)

From the results of the above Activity, we can conclude that the relative complement of sets is neither commutative nor associative.

ii The complement of a set

Let U = {all human beings} and F = {all females}

The Venn diagram of these two sets is as shown. The yellow shaded region (in **U** but outside F) is called the complement of F, denoted by F'.

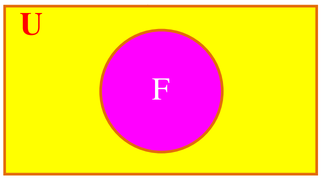


Figure 3.13

It represents all human beings who are not female. The members of F ' are all those members of U that are not members of F.

Definition 3.12

Let A be a subset of a universal set U. The complement (or absolute complement) of A, denoted by A', is defined to be the set of all elements of U that are not in A.

i.e., A' = {x | x∈ U and x ∉ A}.

Using a Venn diagram, we can represent A' by the shaded region as shown in Figure 3.14.

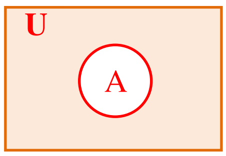


Figure 3.14

Note that for any set A and universal set U,

A′ = U\A

Example 6 In copies of the Venn diagram on the right, shade

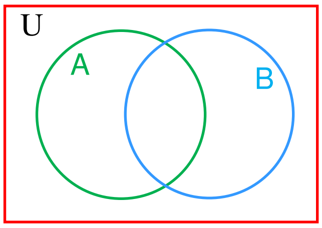


Figure 3.15

a A\B **b** (A⋂B)'

c A⋂B' **d** A'⋃B'

Solution:

a Since A\B is the set of all elements in A that are not in B, we shade the region in A that is not part of B (*shaded in green in* Figure 3.16).

A – B is shaded

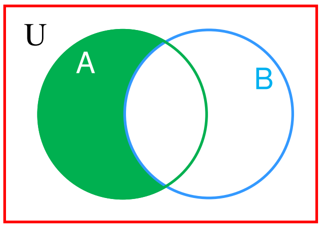


Figure 3.16

b First we shade the region A⋂B; then (A⋂B)' is the region outside A⋂B.

i ii

Figure 3.17

A⋂B is the shaded (*blue*) region. (A⋂B)' is the *green* shaded region.

c First we shade A with strokes that slant upward to the right (////) and shade B' with strokes that slant downward to the right (\\\\).

Then A⋂B' is the cross-hatched region.

i ii

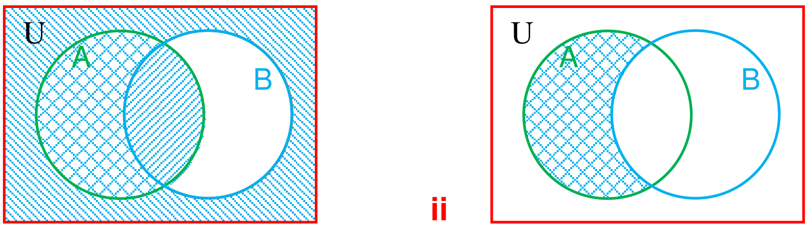


Figure 3.18

A and B' are shaded A⋂B' is shaded

Note that the region of A\B is the same as the region of A⋂B'.

d First we shade A', the region outside A, with strokes that slant upward to the right (////) and then shade B' with strokes that slant downward to the right (\\\\).

Then A'∪B' is the total shaded region.

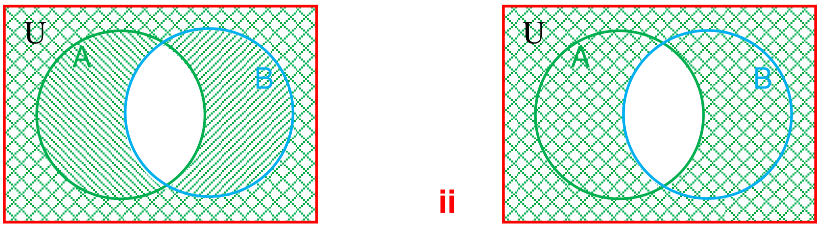


Figure 3.19

A' or B' are shaded   A'∪B' is shaded

Note that the region of (A∩B)' is the same as the region A'∪B'.

Note: When we draw two overlapping circles within a universal set, four regions are formed. Every element of the universal set U is in exactly one of the following regions.

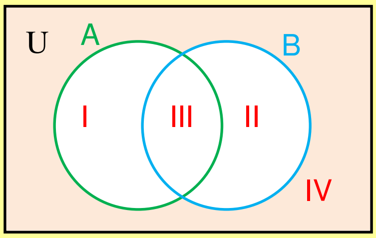


Figure 3.20

- I in A and not in B (A\B)
- II in B and not in A (B\A)
- III in both A and B (A ∩B)
- IV in neither A nor B ((A∪B)')

From Activity 3.13 and the above examples, you generalize as follows:

For any two sets A and B, the following properties hold true:

1 A\B = A ∩ B'   **2** (A ∩ B)' = A' ∪ B'   **3** (A\B)∪B = A∪B

ACTIVITY 3.14

1 In copies of the same Venn diagram used in Example 6, shade

a (A∪B)'   **b** A'∩B'

2 Generalize the result you got from Question 1.

Historical Note:

Augustus De Morgan (1806-1871)

Augustus De Morgan was the first professor of mathematics at University College London and a cofounder of the London Mathematical Society.

De Morgan formulated his laws during his study of symbolic logic. De Morgan's laws have applications in the areas of set theory, mathematical logic and the design of electrical circuits.

Group Work 3.2

1 Copy Figure 3.21 and shade the region that represents each of the following

a $(A \cup B)'$

b $A' \cup B'$

c $(A \cap B)'$

d $A' \cap B'$

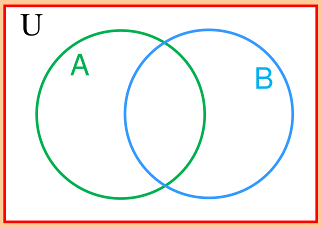


Figure 3.21

2 Discuss what you have observed from Question 1

The above Group Work leads you to the following law called De Morgan's law.

Theorem 3.1 De Morgan's law

For any two sets A and B

1 $(A \cap B)' = A' \cup B'$ 2 $(A \cup B)' = A' \cap B'$

Exercise 3.8

1 Given A = {a, b, c} and B = {b, c, d, e} find:

a the relative complement of A with respect to B.

b the complement of B relative to A.

c the complement of A relative to B.

2 In each of the Venn diagrams given below, shade A\ B.

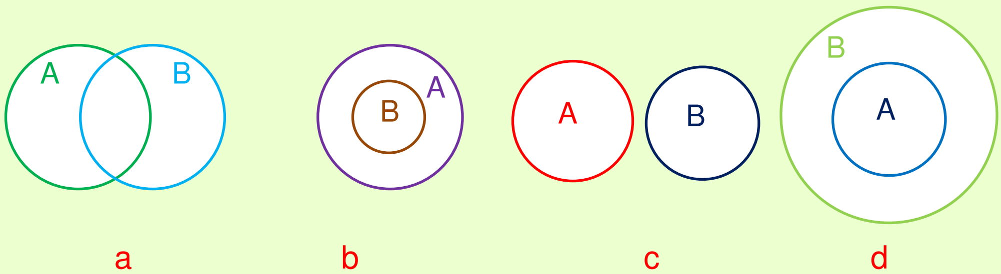


Figure 3.22

3 Determine whether each of the following statements is true or false:

a If $x \in A$ and $x \notin B$ then $x \in (B \setminus A)$

b If $x \in (A \setminus B)$ then $x \in A$

c $B \setminus A \subseteq B$, for any two sets A and B

d $(A\backslash B) \cap (A\cap B) \cap (B\backslash A) = \varnothing$, for any two sets A and B

e If $A\backslash B = \varnothing$ then $A = \varnothing$ and $B = \varnothing$

f If $A \subseteq B$ then $A\backslash B = \varnothing$

g If $A\cap B = \varnothing$ then $(A\backslash B) = A$

h $(A\backslash B) \cup B = A\cup B$, for any two sets A and B

i $A\cap A' = \varnothing$

4 Let $U = \{1, 2, 3, \ldots, 8, 9\}$ be the universal set and $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$ and $C = \{3, 4, 5, 6\}$. List the elements of each of the following:

a A' **b** B' **c** $(A\cup C)'$ **d** $(A\backslash B)'$

e $A'\cap B'$ **f** $(A\cup B)'$ **g** $(A')'$ **h** $B\backslash C$ **i** $B\cap C'$

iii The symmetric difference between two sets

ACTIVITY 3.15

Let $A = \{a, b, d\}$ and $B = \{b, d, e\}$. Then find:

a $A\cap B$ **b** $A\cup B$ **c** $A\backslash B$

d $B\backslash A$ **e** $(A\cup B)\backslash(A\cap B)$ **f** $(A\backslash B) \cup (B\backslash A)$

Compare the results of e and f.

What can you conclude from your answers?

The result of the above Activity leads you to state the following definition.

Definition 3.13

Let A and B be any two sets. The symmetric difference between A and B, denoted by $A\Delta B$, is the set of all elements in $A\cup B$ that are not in $A\cap B$. That is $A\Delta B = \{x \mid x\in (A\cup B) \text{ and } x\notin (A\cap B)\}$

or $A\Delta B = (A\cup B)\backslash(A\cap B)$.

Using a Venn diagram, $A\Delta B$ is illustrated by shading the region in $A\cup B$ that is not part of $A\cap B$ as shown.

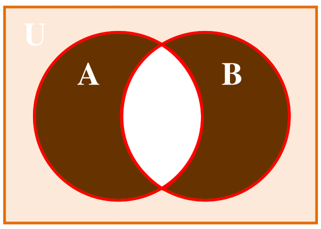


Figure 3.23

$A\Delta B$ is the shaded *dark brown region.*

From Activity 3.15 and the above Venn diagram, you observe that

$A\Delta B = (A\cup B)\backslash (A\cap B) = (A\backslash B) \cup (B\backslash A)$.

Note: If $A \cap B = \varnothing$ then $A\Delta B = A \cup B$.

Example 7 Let A = {–1, 0, 1} and B = {1, 2}. Find AΔB.

Solution: AUB = {–1, 0, 1, 2}; A⋂B = {1 }

∴ AΔB = (AUB)\(A⋂B) = {–1, 0, 2}

Example 8 Let A = {*a*, *b*, *c*} and B = {*d*, *e*}. Find AΔB.

Solution: AUB = { *a*, *b*, *c*, *d*, *e*}; A⋂B = ∅

∴ AΔB = (AUB)\∅ = AUB = {*a*, *b*, *c*, *d*, *e*}

Distributivity

Group Work 3.3

1 Given sets A, B and C, shade the region that represents each of the following

a A⋃ (B⋂ C)

b (AUB) ⋂ (AUC)

c A⋂ (BUC)

d (A⋂ B) ⋃ (A⋂ C)

2 Discuss what you have observed from Question 1.

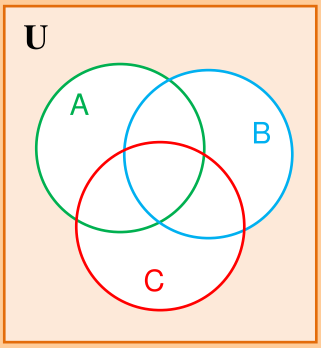


Figure 3.24

As you may have noticed from the above Group Work, the following distributive properties are true:

Distributive properties

For any sets A, B and C

1 Union is distributive over the intersection of sets.

i.e., AU (B⋂C) = (AUB) ⋂ (AUC).

2 Intersection is distributive over the union of sets.

i.e., A⋂(BUC) = (A⋂B) U (A⋂C).

Exercise 3.9

1 If A⋂B = {1, 0, –1} and A⋂C = {0, –1, 2, 3}, then find A⋂(BUC).

2 Simplify each of the following by using Venn diagram or any other property.

a A⋂(AUB) b P'⋂(PUQ)

c A⋂(A'UB) d PU(P⋂Q)

3.3.2 Cartesian Product of Sets

In this subsection, you will learn how to form a new set of ordered pairs from two given sets by taking the Cartesian product of the sets (named after the mathematician Rene Descartes).

Group Work 3.4

A six-sided die (a cube) has its faces marked with numbers 1,2,3,4,5 and 6 respectively.

Two such dice are thrown and the numbers on the resulting upper faces are recorded. For example, (6, 1) means that the number on the upper face of the first die is 6 and that of the second die is 1. We call these ordered pairs, the outcomes of the throw of our dice.

List the set of all possible outcomes of throws of our two dice such that the two numbers:

- i A: are both even.
- ii B: are both odd.
- iii C: are equal.
- iv D: have a sum equal to 8.
- v E: have a sum equal to 14.
- vi F: have an even sum.
- vii G: have the first number 1 and the second number odd.
- viii H: have a sum less than 12.

For example, A= {(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)}.

The activity of this Group Work leads you to learn about the sets whose elements are ordered pairs.

Ordered pair

An *ordered pair* is an element (x, y) formed by taking x from one set and y from another set. In (x, y), we say that x is the first element and y is the second element.

Such a pair is ordered in the sense that (x, y) and (y, x) are not equal unless $x = y$.

Equality of ordered pairs

$(a, b) = (c, d)$, if and only if $a = c$ and $b = d$.

Earlier also we have discussed ordered pairs when we represented points in the Cartesian coordinate plane. A point P in the plane corresponds to an ordered pair (a, b) where a is the x-coordinate and b is the y-coordinate of the point P.

Example 1 A weather bureau recorded hourly temperatures as shown in the following table.

Time	9	10	11	12	1	2	3
Temp	61	62	65	69	68	72	76

This data enables us to make seven sentences of the form:

At x o'clock the temperature was y degrees.

That is, using the ordered pair (x, y),the ordered pair (9, 61) means.

At 9 *o'clock the temperature was* 61 *degrees*.

So the set of ordered pairs {(9, 61), (10, 62), (11, 65), (12, 69), (1, 68), (2, 72), (3, 76)} are another form of the data in the table, where the first element of each pair is time and the second element is the temperature recorded at that time.

Definition 3.14

Given two non-empty sets A and B, the set of all ordered pairs (a, b) where $a \in$ A and $b \in$ B is called the Cartesian product of A and B, denoted by A × B (read "A cross B").

i.e., $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.

Note that the sets A and B in the definition can be the same or different.

Example 2 If A = { 1, 2, 3} and B = { 4, 5}, then

A × B = {(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)}

Example 3 Let A = { a, b}, then form A × A.

Solution: $A \times A = \{(a, a), (a, b), (b, a), (b, b)\}$.

Example 4 Let A = {–1, 0} and B = {–1, 0, 1}.

Find A × B and illustrate it by means of a diagram.

Solution: A × B = {(–1, –1), (–1, 0), (–1, 1), (0, –1), (0, 0), (0, 1)}

The diagram is as shown in Figure 3.25.

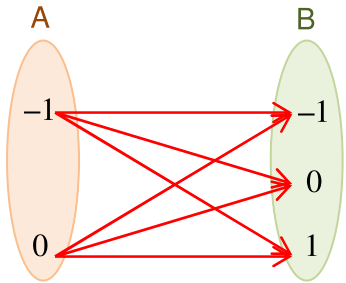
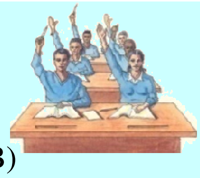


Figure 3.25

Note: $n(A \times B) = n(A) \times n(B)$.

ACTIVITY 3.16



1 Let A = {2, 3} and B = {0, 1, 2}. Find:

a $A \times B$ **b** $B \times A$ **c** $n(A \times B)$

2 Let A = {*a*, *b*} B = {*c*, *d*, *e*} and C = {*f*, *e*, *c*}. Find:

a $A \times (B \cap C)$ **b** $A \times (B \cup C)$

c $(A \times B) \cap (A \times C)$ **d** $(A \times B) \cup (A \times C)$

From the result of the Activity, you conclude that:

For any sets A, B and C

i $A \times B \neq B \times A$, for $A \neq B$ *Cartesian product of sets is not commutative.*

ii $n(A \times B) = n(A) \times n(B) = n(B \times A)$. *where A and B are finite sets.*

iii $A \times (B \cap C) = (A \times B) \cap (A \times C)$. *Cartesian product is distributive over intersection.*

iv $A \times (B \cup C) = (A \times B) \cup (A \times C)$. *Cartesian product is distributive over union.*

Exercise 3.10

1 Given A = {2} B = {1, 5} C = {−1, 1} find:

a $A \times B$ **b** $B \times A$ **c** $B \times C$ **d** $A \times (B \cap C)$

e $(A \cup C) \times B$ **f** $(A \times B) \cup (A \times C)$ **g** $B \times B$

2 If $B \times C$ = {(1, 1), (1, 2), (1, 3), (4, 1), (4, 2), (4, 3)}, find:

a B **b** C **c** $C \times B$

3 If $n(A \times B) = 18$ and $n(A) = 3$ then find $n(B)$.

4 Let U = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9} be the universal set and A = {0, 2, 4, 6, 8, 9}, B = {1, 3, 6, 8} and C = {0, 2, 3, 4, 5}. Find:

a $A' \times C'$ **b** $B \times A'$ **c** $B \times (A' \setminus C)$

5 If $(2x + 3, 7) = (7, 3y + 1)$, find the values of x and y.

3.3.3 Problems Involving Sets

In this subsection, you will learn how to solve problems that involve sets, in particular the numbers of elements in sets.

The number of elements that are either in set A or set B, denoted by $n(A \cup B)$, may not necessarily be $n(A) + n(B)$ as we can see in the Figure 3.26.

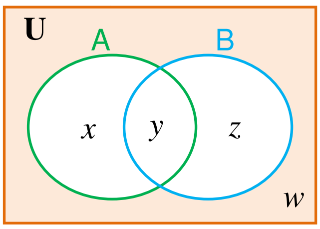


Figure 3.26

In this figure, suppose the number of elements in the closed regions of the Venn diagram are denoted by x, y, z and w.

n (A) = $x + y$ and n (B) = $y + z$.

So, n (A) + n (B) = $x + y + y + z$.

n (A ∪B) = $x + y + z$ = n (A) +n (B) −y

i.e., n (A∪B)= n (A) + n (B) – n (A∩B).

Number of elements in (A∪B)

For any finite sets A and B, the number of elements that are in A∪B is

n (A∪B) = n (A) + n (B) – n (A∩B).

Note: If A∩B =∅, then n (A∪B) = n (A) + n (B).

Example 1 Explain why n (A – B) = n (A) – n (A∩B).

Solution: From Figure 3.26 above, n (A) = $x + y$, n (A∩B) = y

n (A) – n (A∩B) = $(x + y) – y = x$,

x is the number of elements in A that are not in B. So, n (A – B) = x.

∴ n (A – B) = x = n (A) – n (A∩B).

For any finite sets A and B,

n (A\B) = n (A) – n (A∩B)

Example 2 Among 1500 students in a school, 13 students failed in English, 12 students failed in mathematics and 7 students failed in both English and Mathematics.

i How many students failed in either English or in Mathematics?

ii How many students passed both in English and in Mathematics?

Solution: Let E be the set of students who failed in English, M be the set of students who failed in mathematics and U be the set of all students in the school.

Then, n (E) = 13, n (M) = 12, n (E∩M) = 7 and n (U) = 1500.

i n (E∪M) = n (E) + n (M) – n (E∩M) = 13 + 12 – 7 = 18.

ii The set of all students who passed in both subjects is U\(E∪M).

n (U\(E∪M)) = n (U) – n (E∪M)= 1500 – 18 = 1482.

Exercise 3.11

1 For A = {2, 3, . . . 6} and B = {6, 7, . . . 10} show that:

a $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ **b** $n(A \times B) = n(A) \times n(B)$

c $n(A \times A) = n(A) \times n(A)$

2 If $n(C \cap D) = 8$ and $n(C \backslash D) = 6$ then find $n(C)$.

3 Using a Venn diagram, or a formula, answer each of the following:

a Given $n(Q \backslash P) = 4$, $n(P \backslash Q) = 5$ and $n(P) = 7$ find $n(Q)$.

b If $n(R' \cap S') + n(R' \cap S) = 3$, $n(R \cap S) = 4$ and $n(S' \cap R) = 7$, find $n(U)$.

4 Indicate whether the statements below are true or false for all finite sets A and B. If a statement is false give a counter example.

a $n(A \cup B) = n(A) + n(B)$ **b** $n(A \cap B) = n(A) - n(B)$

c If $n(A) = n(B)$ then A = B **d** If A = B then $n(A) = n(B)$

e $n(A \times B) = n(A) \cdot n(B)$ **f** $n(A) + n(B) = n(A \cup B) - n(A \cap B)$

g $n(A' \cup B') = n((A \cup B)')$ **h** $n(A \cap B) = n(A \cup B) - n(A \cap B') - n(A' \cap B)$

i $n(A) + n(A') = n(U)$

5 Suppose A and B are sets such that $n(A) = 10$, $n(B) = 23$ and $n(A \cap B) = 4$, then find:

a $n(A \cup B)$ **b** $n(A \backslash B)$ **c** $n(A \Delta B)$ **d** $n(B \backslash A)$

6 If $A = \{x \mid x$ is a non-negative integer and $x^3 = x\}$, then how many proper subsets does A have?

7 Of 100 students, 65 are members of a mathematics club and 40 are members of a physics club. If 10 are members of neither club, then how many students are members of:

a both clubs? **b** only the mathematics club?

c only the physics club?

8 The following Venn diagram shows two sets A and B. If $n(A) = 13$, $n(B) = 8$, then find:

a $n(A \cup B)$ **b** $n(U)$

c $n(B \backslash A)$ **d** $n(A \cap B')$

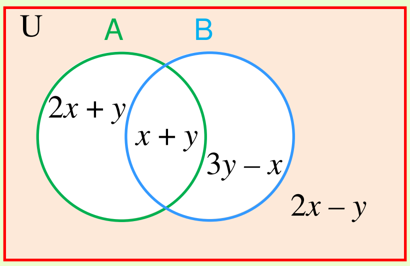


Figure 3.27

Key Terms

complement	infinite set	set
disjoint sets	intersection of sets	subset
element	power set	symmetric difference between sets
empty set	proper subset	union of sets
finite set	relative complement	universal set

Summary

1. A set is a well-defined collection of objects. The objects of a set are called its elements (or members).
2. Sets are described in the following ways:
   - a Verbal method
   - b Listing method
     - i Partial listing method
     - ii Complete listing method
   - c Set-builder method
3. The universal set is a set that contains all elements under consideration in a discussion.
4. The complement of a set A is the set of all elements that are found in the universal set but not in A.
5. A set S is called finite if and only if it is the empty set or has exactly n elements, where n is a natural number. Otherwise, it is called infinite.
6. A set A is a subset of B if and only if each element of A is in set B.
7. i P (A), the power set of a set A, is the set of all subsets of A.

   ii If n (A) = n, then the number of subsets of A is 2^n.
8. Two sets A and B are said to be equal if and only if $A \subseteq B$ and $B \subseteq A$.
9. Two sets A and B are said to be equivalent if and only if there is a one-to-one correspondence between their elements.
10. i A set A is a proper subset of set B, denoted by $A \subset B$, if and only if $A \subseteq B$ and $B \not\subseteq A$.

    ii If n (A) = n, then the number of proper subsets of A is $2^n - 1$.

11 Operations on sets; for any sets A and B,

- **i** $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.
- **ii** $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.
- **iii** $A - B$ (or $A \backslash B$) $= \{x \mid x \in A \text{ and } x \notin B\}$.
- **iv** $A \Delta B = \{x \mid x \in (A \cup B) \text{ and } x \notin (A \cap B)\}$.
- **v** $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.

12 Properties of union, intersection, symmetric difference and Cartesian product:

For all sets A, B and C:

- **i** Commutative properties
  - **a** $A \cup B = B \cup A$ **b** $A \cap B = B \cap A$ **c** $A \Delta B = B \Delta A$
- **ii** Associative properties
  - **a** $A \cup (B \cup C) = (A \cup B) \cup C$ **c** $A \Delta (B \Delta C) = (A \Delta B) \Delta C$
  - **b** $A \cap (B \cap C) = (A \cap B) \cap C$
- **iii** Identity properties
  - **a** $A \cup \varnothing = A$ **b** $A \cap U = A$ (U is a universal set)
- **iv** Distributive properties
  - **a** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
  - **b** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  - **c** $A \times (B \cup C) = (A \times B) \cup (A \times C)$
  - **d** $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- **v** De Morgan's Law
  - **a** $(A \cup B)' = A' \cap B'$ **b** $(A \cap B)' = A' \cup B'$
- **vi** For any set A
  - **a** $A \cup A' = U$ **b** $(A')' = A$
  - **c** $A \cap A' = \varnothing$ **d** $A \times \varnothing = \varnothing$

Review Exercises on Unit 3

1 Which of the following are sets?

- **a** The collection of all tall students in your class.
- **b** The collection of all natural numbers divisible by 3.
- **c** The collection of all students in your school.
- **d** The collection of all intelligent students in Ethiopia.
- **e** The collection of all subsets of the set {1, 2, 3, 4, 5}.

2 Rewrite the following statements, using the correct notation:

a B is a set whose elements are x, y, z and w.

b 3 is not an element of set B.

c D is the set of all rational numbers between $\sqrt{2}$ and $\sqrt{5}$.

d H is the set of all positive multiples of 3.

3 Which of the following pairs of sets are equivalent?

a $\{1, 2, 3, 4, 5\}$ and $\{m, n, o, p, q\}$

b $\{x \mid x$ is a letter in the word mathematics$\}$ and $\{y \in \mathbb{N} \mid 1 \le y \le 11\}$

c $\{a, b, c, d, e, f, \ldots\ m\}$ and $\{1, 2, 3, 4, 5, \ldots 13\}$

4 Which of the following represent equal sets?

$A = \{a, b, c, d\}$ $B = \{x, y, z, w\}$

$C = \{x \mid x$ is one of the first four letters in the English alphabet$\}$

$D = \varnothing$ $E = \{0\}$ $F = \{x \mid x \neq x\}$ $G = \{x \in \mathbb{Z} \mid -1 < x < 1\}$

5 If $U = \{a, b, c, d, e, f, g, h\}$, $A = \{b, d, f, h\}$ and $B = \{a, b, e, f, g, h\}$, find the following:

a A' **b** B' **c** $A \cap B$ **d** $(A \cap B)'$ **e** $A' \cap B'$

6 In the Venn diagram given below, write the region labelled by I, II, III and IV in terms of A and B.

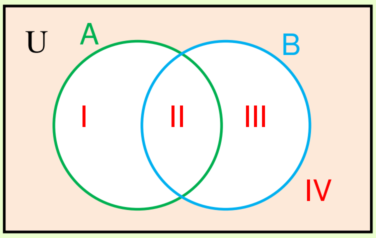


Figure 3.28

7 For each of Questions a, b and c, copy the following Venn diagram and shade the regions that represent:

a $A \cap (B \cap C)$. **b** $A \backslash (B \cap C)$.

c $A \cup (B \backslash C)$.

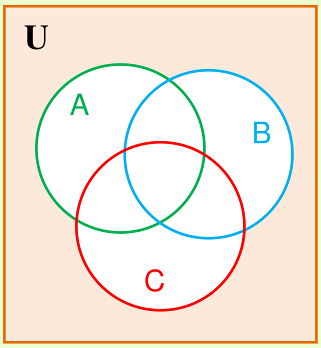


Figure 3.29

8 Let $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4, 5\}$, $B = \{0, 2, 1, 6, 8\}$ and $C = \{3, 6, 9\}$. Then find:

a A' **b** $B \backslash A$ **c** $A \cap C'$

d $C \times (A \cap B)$ **e** $(B \backslash A) \times C$

9 Suppose B is a proper subset of C,

a If n (C) = 8, what is the maximum number of elements in B?

b What is the least possible number of elements in B?

10 If n (U) = 16, n (A) = 7 and n (B) = 12, find:

a n (A') **b** n (B')

c greatest n (A∩B) **d** least n (A∪B)

11 In a class of 31 students, 22 students study physics, 20 students study chemistry and 5 students study neither. Calculate the number of students who study both subjects.

12 Suppose A and B are sets such that A∪B has 20 elements, A∩B has 7 elements, and the number of elements in B is twice that of A. What is the number of elements in:

a A? **b** B?

13 State whether each of the following is finite or infinite:

a $\{x \mid x$ is an integer less than 5$\}$

b $\{x \mid x$ is a rational number between 0 and 1$\}$

c $\{x \mid x$ is the number of points on a 1 cm-long line segment$\}$

d The set of trees found in Addis Ababa.

e The set of "teff" in 1,000 quintals.

f The set of students in this class who are 10 years old.

14 How many letters in the English alphabet precede the letter v? (Think of a shortcut method).

15 Of 100 staff members of a school, 48 drink coffee, 25 drink both tea and coffee and everyone drinks either coffee or tea. How many staff members drink tea?

16 Given that set A has 15 elements and set B has 12 elements, determine each of the following:

a The maximum possible number of elements in A∪B.

b The minimum possible number of elements in A∪B.

c The maximum possible number of elements in A∩B.

d The minimum possible number of elements in A∩B.